Simulation of the soil-water characteristic curve using laboratory tests and empirical models for Cubans unsaturated soils

Simulación de la curva característica del suelo empleando ensayos de laboratorio y modelos empíricos para suelos no saturados cubanos

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Abstract
The task of characterizing an unsaturated soil is a very hard one, unlike the classics hypothesis like 100% of saturation are not considerable now. This work focuses on obtaining and simulating the soil-water characteristic curve for unsaturated soils from: Sagua la Grande, Formation Capdevila, Cienfuegos and Mariel, located in four provinces of Cuba. The estimation of the characteristic soil has different purposes that can be used to obtain various parameters which are important in the description of unsaturated soils. The permeability, shear strength and volumetric behavior are soil properties that are affected by the suction variation. For each soil case study, the characteristic curve is obtained using the filter paper test laboratory. The simulation is performed through Van Genuchten, Fredlund & Xing, and Gallipoli methods, that are empirical models and the curve is hyperbolic shaped. The statistical analysis helped to determine which of the models fit the properties of soils described by the experimental values.

Keywords: soil-water characteristic curve, unsaturated soils, suction, degree of saturation, simulation.

Resumen
En el trabajo se presenta la obtención y simulación de la curva característica, para cuatro suelos no saturados, pertenecientes a las localidades de Sagua la Grande, la Formación Capdevila, Cienfuegos y el Mariel ubicadas en cuatro provincias de Cuba. La estimación de la curva característica del suelo tiene diversos intereses pudiéndose emplear para obtener varios parámetros utilizados en la descripción de los suelos no saturados. La permeabilidad, la resistencia al cortante así como el comportamiento volumétrico son propiedades del suelo que se ven afectadas por la variación de la succión. Para cada suelo de estudio se obtienen la curva característica empleando el ensayo de laboratorio del papel de filtro, y la simulación se realizará mediante la utilización de métodos de Van Genuchten, Fredlund & Xing, y Gallipoli, que son modelos empíricos y la curva es de tipo hiperbólica, determinando mediante análisis estadísticos cual es el modelo que se ajusta a los valores experimentales.

Palabras Claves: Curva de característica del suelo, suelo no saturado, succión, grado de saturación, simulación.

The use of natural soil as a building material is known since the beginning of times (Barros & Imhoff, 2010). Soil mechanics was initially developed to study the saturated soils. However, much of the geological and man-made structures, including land dams, embankments and earthworks will find formations that are composed of unsaturated soils. For the simplicity it was established to consider the application of the mechanical principles of saturated soils, when the degree of saturation of a ground is greater than 85%. However, when the degree of saturation is less than 85%, it is necessary to apply the principles of mechanics unsaturated soils (Mohamed et al. 2006).

Previous papers (Fredlund & Xing, 1994), (Bicalho et al. 2011) and (Maaitah, 2012) established suction as a key issue in the study of unsaturated soils, because this affects other soil properties directly or indirectly; suction undergoes variations with climate change.

Working with unsaturated soils is a complex assignment, an important role plays the characterization of its mechanical properties, the most important aspect of the survey is the knowledge of the characteristic curve of the soil, since the variation of the suction affects permeability, shear strength, load capacity and the volumetric behavior. Therefore it is proposed in this work to deepen the state of knowledge of the behavior of the suction of unsaturated soils, by obtaining the characteristic curve of the soil and simulate using empirical methods.

State of Art

For (Fredlund & Rahardjo, 1993), (Padilla, 2010) and (Alanís, 2012), the effect of the suction in unsaturated soil is equal to external pressure applied. This suction is considered comprising two addends and their values can range from 0 kPa to 1 GPa. The total suction experienced by a soil can be stated as:

$$S = S_m + S_o \qquad (1)$$

where:

S_m, matric suction is the negative pressure of interstitial water, the suction is directly related to the state of stress resulting from surface phenomena and gravitational (($u_a - u_w$), where u_a, is the pressure of air pores and u_w is the pore water pressure). The value of this matrix suction depends on the surface strain and the radius of curvature of the meniscus. When the degree of saturation decreases, the meniscus is retracted in small pore spaces where the radius of curvature of the meniscus is reduced and thus increases the matrix suction.

S_o, osmotic suction is a negative pressure of pure water which must be subjected to a body of water with the same composition as the pore, to be balanced through a half permeable membrane. This suction will be related to the osmotic pressure from water composition.
S, total suction, is the sum of the matrix and the osmotic suction.

The soil suction is directly related to the capacity of water absorption. For the same pore index and varying the humidity or the degree of saturation, will be registered lower or higher suction.

In Cuba the issue of unsaturated soil suction has not been researched so far, although some authors have covered the topic of unsaturated soils but focusing on characterizing the behavior of expansive soils, these works are referenced by (Ábalo & Moya, 1982), (Delgado, 2003) and (Delgado & Quevedo 2009) he suction in the soil can be represented by the soil-water characteristic curve of the ground, this is defined as the relationship between the humidity and the degree of saturation and the suction (Azam & Ito 2011) and (Alanís, 2012), where the degree of saturation is a percentage of the volume of water respect to the volume of voids in a portion of soil and soil suction is the property of the soil to retain water.

The amount of water in the soil-water characteristic curve can be expressed by the water content (ω), by the degree of saturation (S_r), or the void ratio occupied by water (e_w). Proper representation of the curve has a consequence in the characterization of soil physical behavior, because that indirectly relates the mechanical behavior of unsaturated soils. (Otálvaro, 2013)

The suction values match to soil type, with a certain density, and the nature of that relationship is directly bonded to the particle size and soil mineralogy. In general, the pore geometry, size and mineralogical composition of the fine fraction are determining the relative position, shape, and slope of the curve. (Fredlund & Xing, 1994)

The characteristic curve of the soil can be described from two typical trajectories in drying or wetting. Wetting curve, the end point of the curve may differ from the wetting point of the drying curve because of air entrapment in the soil. In the second curve, the drying process is first produced in the bigger pores, then the connectors and the smaller pores of the soil, while the curve of wetting occurs in

the opposite direction, wetting occurs at the connectors first and smaller pores, then larger pores, which show a difference in the shapes of the two curves, which leads to the phenomenon of hysteresis.

The hysteresis is likely to occur for several reasons, such as the lack of geometrical uniformity of the individual pores, the contact angle effect, encapsulation of air pores "end", which reduces the moisture content of a fresh ground and moistened soil volume change due to the history, weather, expansion or shrinkage. (Hillel, 1998). Over time various models have been proposed for estimating the characteristic of soils from field observations and laboratory building relationships through linear and nonlinear approaches.

Empirical models or strictly mathematical equations aim to capture the particularities of the soil-water characteristic curve, including its shape. The mathematical representation which is the characteristic curve is usually hyperbolic type. (Otálvaro, 2013), (Matlan et al. 2014). In formulations obtained to estimate the characteristic of the soils are:

Method of Van Genuchten (1980)
Van Genuchten model assumes that the main drying curves retention and wetting can be described accurately by the expression: (Matlan et al. 2014)

$$S_r = (|1 + |\alpha S|^\eta)^{-m}, \ S < 0 \text{ and } S_r = 1, \ S \geq 0 \quad\quad (2)$$

Where, S_r is the degree of saturation, S is the total suction of the sample, α and η represent the fitting parameters of $m = 1 - 1/\eta$ method.

Method of Williams (1983)
The model of Williams, in turn means that the main drying curves retention and wetting can be described by the following expression. (Alanís, 2012)

$$\ln S = a_1 + b_1 \ln \omega \quad\quad (3)$$

Where the parameter S is the total suction, a_1 and b_1 are curve fitting parameters and ω is the humidity value for that value of suction.

Method Fredlund and Xing (1994)
A fitting equation was proposed by (Fredlund & Xing, 1994) for obtaining main wetting and drying curves retaining described by the expression:

$$\Theta = C(S) \frac{\theta_s}{[\ln[e + (S/a)^\eta]]^m} \quad\quad (4)$$

$$C(S) = \frac{\ln(1 + S/S_{res})}{\ln[1 + (1000000/S_{res})]} \qu\quad (5)$$

Θ where the parameter that represents the normalized water content, S is the total suction experienced by the soil, a, m, η are fitting parameters of the model, e is the natural number Euler; 2.7182 and S_{res} is corresponding to the residual moisture content of suction.

Method of Gallipoli (2003)
A proposal for a modified expression of the soil characteristic curve given by Van Genuchten is raised by (Gallipoli et al. 2003), in which the degree of saturation depends not only sucking but also the volume soil unit. The expression is given below:

$$S_r = \left\{ 1 + [\phi(\nu - 1)^\psi * s]^n \right\}^{-m} \quad\quad (6)$$

Where, ϕ, ψ, n and m are fitting parameters of the model and ν, is the unit volume of soil that is equal to ν = 1 + e, where "e" is the void ratio.

Different formulations are analyzed empirical models to simulate the characteristic curves of unsaturated soils. Formulations Van Genuchten, Fredlund and Xing, and Gallipoli are the most widely accepted for the simulation of the characteristic curves of unsaturated soils, because it has a hyperbolic shape and does represent the hysteresis that is experienced in the field of drying and wetting.

Experimental Results

In order to describe the studied soils, properties like: particle size, specific gravity, liquid limit, plastic limit and plasticity index, Proctor Standard and Proctor Modified; where determined by preforming several tests following the guidelines of the existing ASTM also suction in soils was found. The results of physical tests performed on soil samples are shown below.

Particle size distribution, consistency limits and specific weight
The results of the particle size, consistency limits and the specific gravity of the solid particles of the soil samples under study are shown below in figure 1 and table 1.

Figure 1 Average curves of the samples grain size of the soils studied. Source: own elaboration, 2013
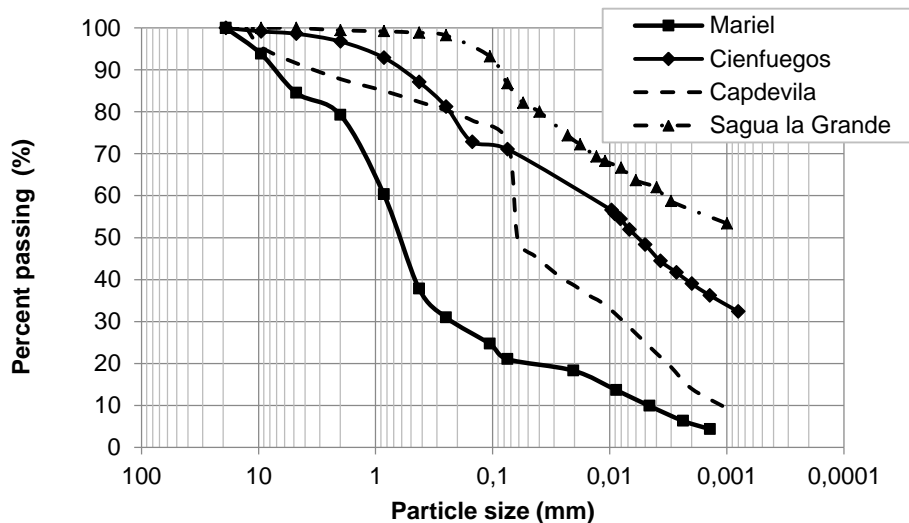

Table 1 Consistency limits and specific gravity of the solid particles, of the soil samples tested. Source: own elaboration, 2013

Parameters	Capdevila	Sagua la Grande	Cienfuegos	Mariel
Liquid Limit (LL)	52	95	42	34
Plastic Limit (PL)	21	25	21	21
Plasticity Index (PI)	31	70	21	13
Specific gravity of solids (Gs)	2,72	2,74	2,7	2,7

Depending on the results of the particle size and consistency limits soils studied soils are classified of USCS method.
-   Capdevila, CH (clay of high compressibility)
-   Sagua la Grande, CH (clay of high compressibility)
-   Cienfuegos, CL clay of low compressibility)
-   Mariel, SC (loamy sand with gravel)

Proctor Standard and Modified Test
Proctor Standard test was performed to Capdevila, Sagua la Grande and Cienfuegos soil samples; and the Modified Proctor test to the Mariel sample necessary to obtain the values of maximum dry unit weight and optimum moisture to have a reference when working on remolded samples because they could not be obtained undisturbed samples.

The remolding of the sample used in the research was done following the criteria of maximum dry unit weight, always working on the drying branch with a variation of 3% of moisture with respect to the optimum moisture of the Proctor curve obtained in each case. This test was performed taking into account the standard and the values obtained are shown in figure 2.

The Proctor test was performed in each case with the initial moisture sampling, as the pre-drying of the soil can affect behavior, especially with regard to the conditions adopted in the field where practice is not realized.

Figure 2 Curves (γ_d vs ω) average for the Standard and Modified Proctor energy of the soil samples studied. Source: own elaboration, 2013
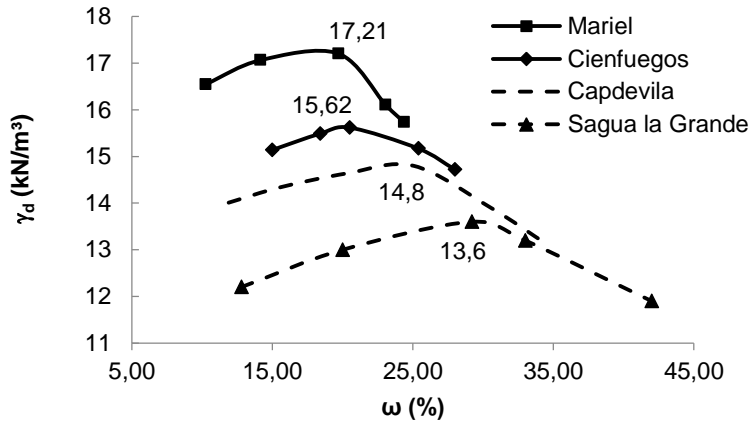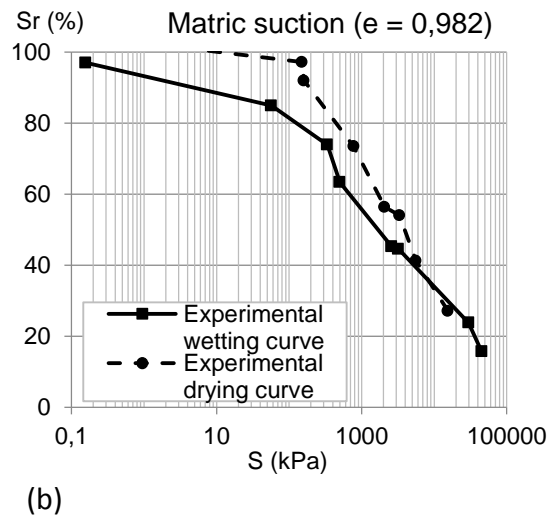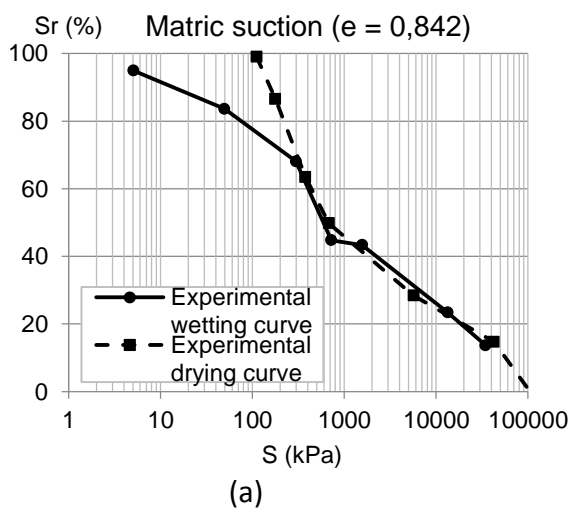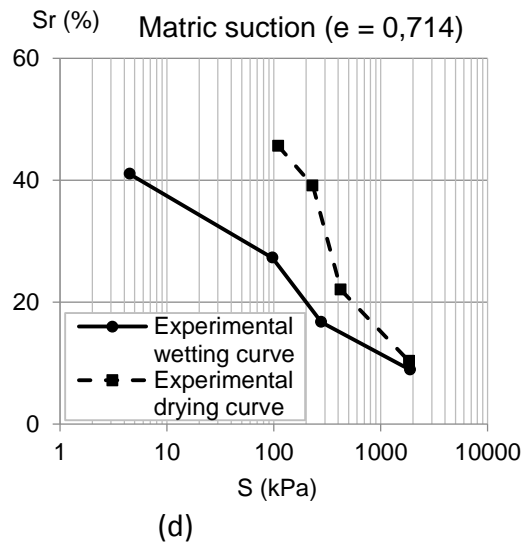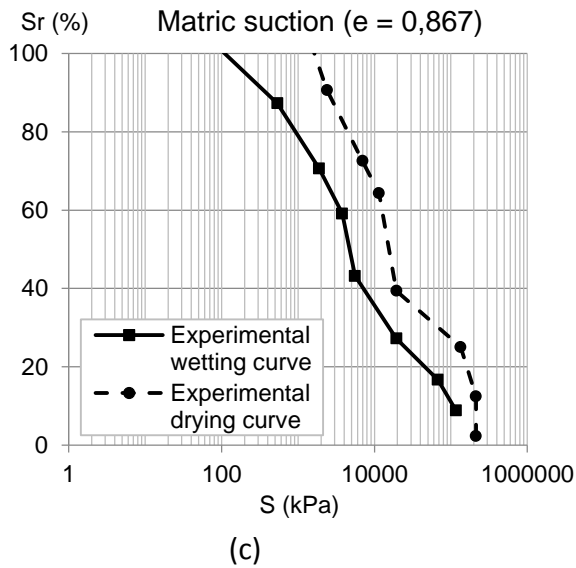

The soil-water characteristic curve of the soils studied

To determine soil suction, was taken as the standard basis, experimental results suctions of the soil samples used in the research are presented below in figure 3. The suction test was performed using Whatman filter paper 42.

When performing the suction test remoulded samples were used, taking into account the specific weight and moisture obtained by Proctor test, due to not having natural samples of soil studied.

Figure 3 The experimental soil-water characteristics curves of (a) Capdevila soils, (b) Sagua la Grande soils, (c) Cienfuegos soils and (d) Mariel soils. Source: own elaboration, 2013
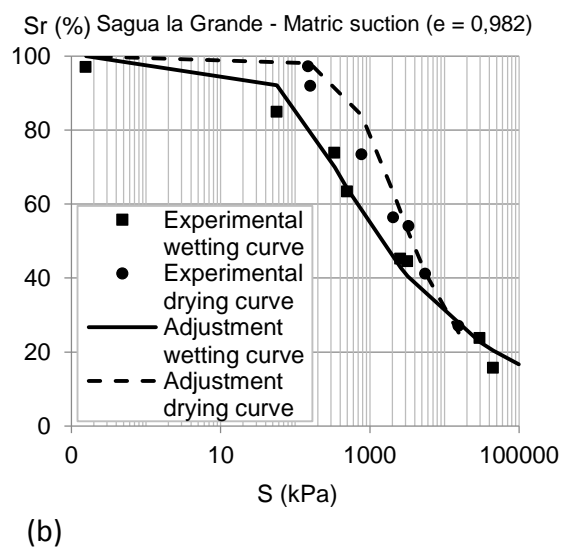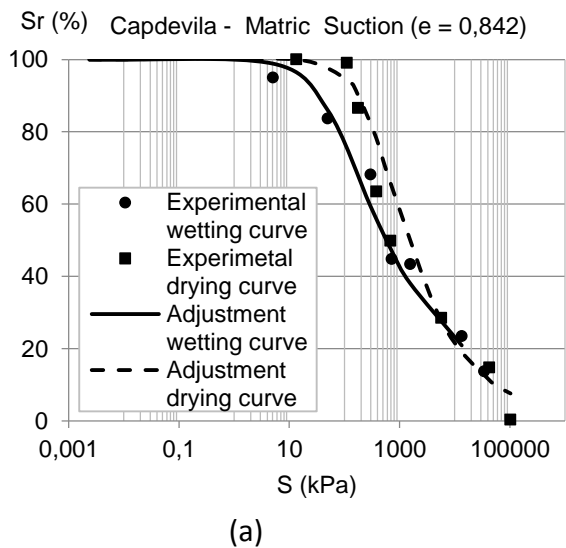
(c)

(d)

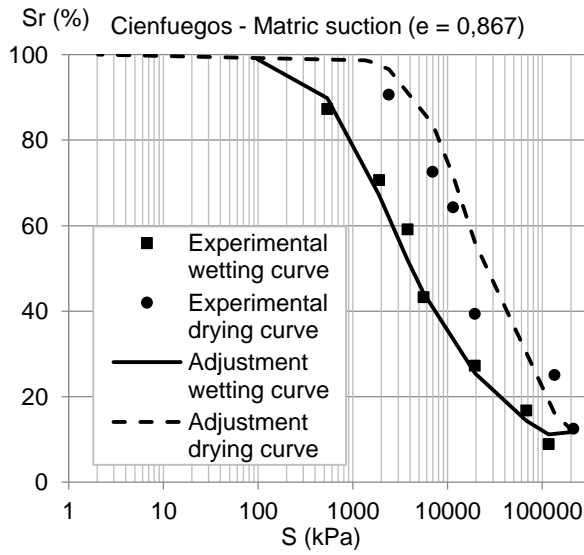**Obtaining the soil-water characteristic curve, Method of Van Genuchten**

Employing equation 2 and setting parameters α, η and m = 1-1/η, adjusting the soils-water characteristic curve for soils studied is determined as shown in figure 4. The values of parameters model to appear in table 2.

Table 2 Adjustment parameters in wetting and drying of Van Genuchten model for soil research. Source: own elaboration, 2013

Soils	Adjustment values in wetting			Adjustment values in drying		
	α	m	η	α	m	η
Capdevila	0,02	0,219	1,28	0,003	0,31	1,45
Sagua la Grande	0,01	0,206	1,26	0,001	0,33	1,5
Mariel	0,05	0,2	1,25	0,04	0,33	1,5
Cienfuegos	0,001	0,315	1,46	0,0001	0,412	1,7

Figure 4 Adjustment the soil-water characteristic curves of (a) Capdevila soils, (b) Sagua la Grande soils, (c) Cienfuegos soils and (d) Mariel soils, by Van Genuchten method. Source: own elaboration, 2013

(a)

(b)

27

Sr (%) Cienfuegos - Matric suction (e = 0,867)

- Experimental wetting curve
- Experimental drying curve
- Adjustment wetting curve
- Adjustment drying curve

(c)

Sr (%) Mariel - Matric suction (e = 0,714)

- Experimental wetting curve
- Experimental drying curve
- Adjustment wetting curve
- Adjustment drying curve

(d)

Obtaining the soil-water characteristic curve, Method of Fredlund

Using equations 4, 5 and adjusting parameters α, η, m, the setting of the soil-water characteristic curve for the soils studied as shown in figure 5, the values of the model parameters are determined to appear in the table 3.

Table 3 Adjustment parameters in wetting and drying of Fredlund model for soil research. Source: own elaboration, 2013
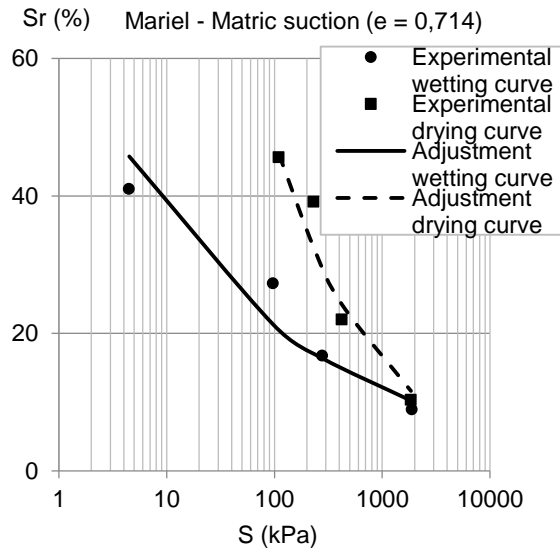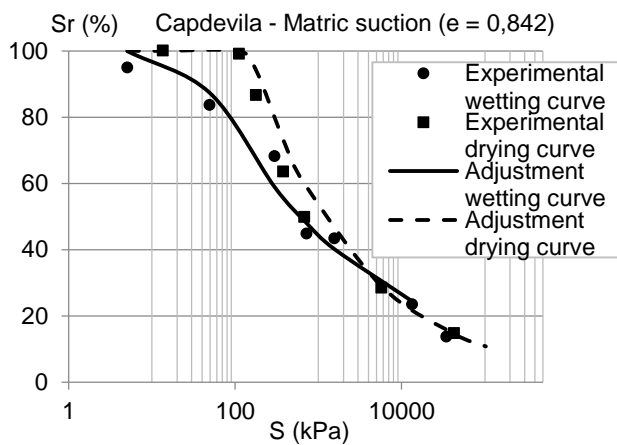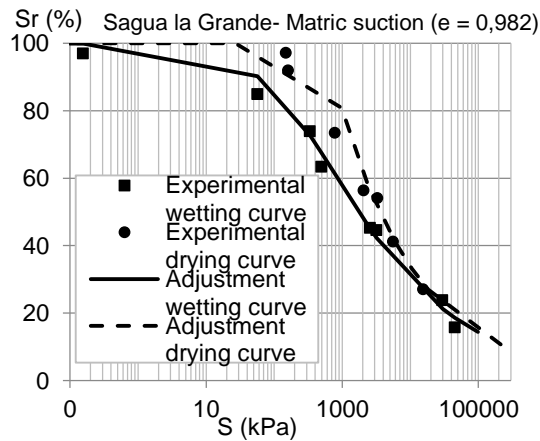
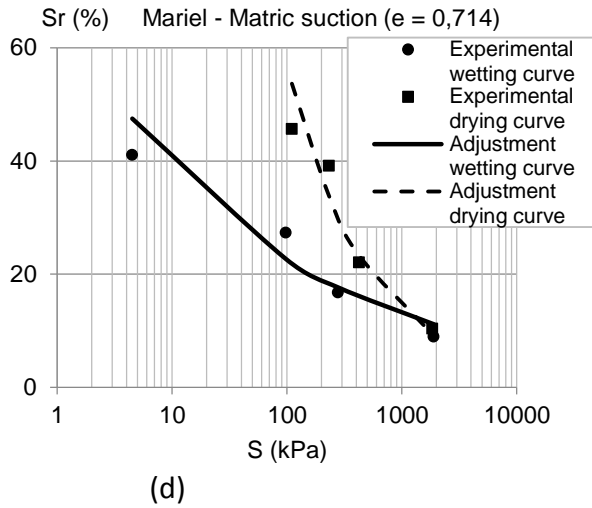| Soils	Adjustment values in wetting			Adjustment values in drying		
	α	m	η	α	m	η
Capdevila	0,3	2,6	0,089	1,5	11	0,031
Sagua la Grande	2,7	0,8	0,41	5,5	7,5	0,05
Mariel	0,002	3	0,08	0,4	20	0,031
Cienfuegos	20	1,1	0,5	22	7,5	0,05

Figure 5 Adjustment the soil-water characteristic curves of Capdevila soils, (a) Capdevila soils, (b) Sagua la Grande soils, (c) Cienfuegos soils and (d) Mariel soils, by Fredlund method. Source: own elaboration, 2013

Sr (%) Capdevila - Matric suction (e = 0,842)

- Experimental wetting curve
- Experimental drying curve
- Adjustment wetting curve
- Adjustment drying curve

(a)

Sr (%) Sagua la Grande- Matric suction (e = 0,982)

- Experimental wetting curve
- Experimental drying curve
- Adjustment wetting curve
- Adjustment drying curve

(b)

28

Sr (%) Cienfuegos - Matric suction (e = 0,867)

(c)

Sr (%) Mariel - Matric suction (e = 0,714)

(d)

Obtaining the soil-water characteristic curve, Method of Gallipoli

Using equations 6 and adjusting the parameters φ, ψ, n and m, the setting of the characteristic curve for the soils studied is determined as shown in figure 6, the values of the model parameters to appear in table 4.

Table 4 Adjustment parameters in wetting and drying of Gallipoli model for soil research. Source: own elaboration, 2013

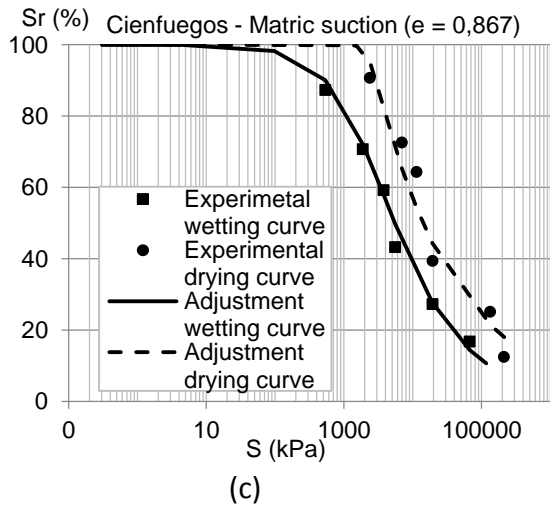| Soils	Adjustment values in wetting				Adjustment values in drying			
	ψ	φ	η	m	ψ	φ	η	m
Capdevila	10	0,012	1,021	0,3	10	0,008	6	0,058
Sagua la Grande	8	550	0,5	0,9	8	0,0004	0,9	0,8
Mariel	10	2,5	0,9	0,3	10	0,06	7	0,058
Cienfuegos	8	750	1,1	0,9	8	0,0001	1,3	0,5

Figure 6 Adjustment the soil-water characteristic curves of (a) Capdevila soils, (b) Sagua la Grande soils, (c) Cienfuegos soils and (d) Mariel soils, by Gallipoli method. Source: own elaboration, 2013

Sr (%) Capdevila - Matric suction (e = 0,842)

(a)

Sr (%) Sagua la Grande- Matric suction (e = 0,982)

(b)

29

(c)

(d)
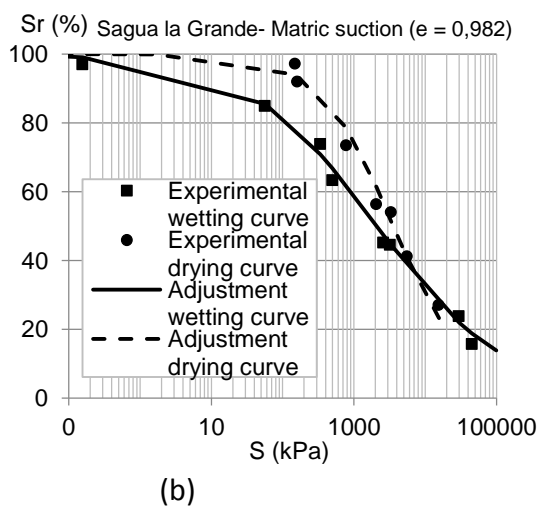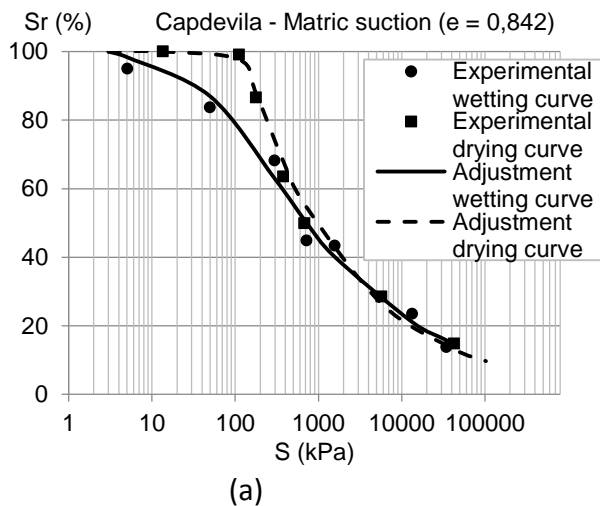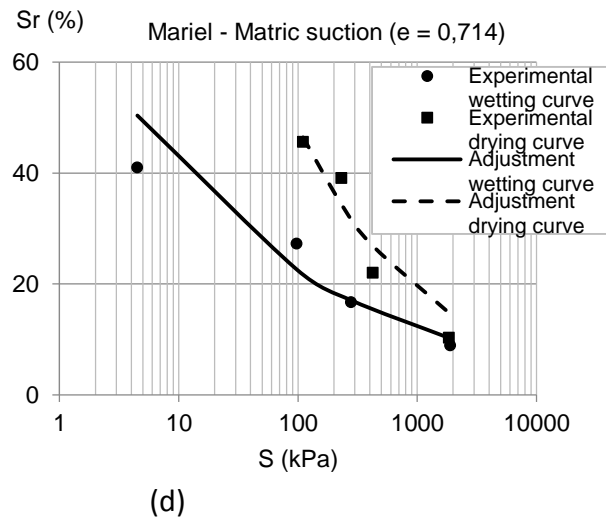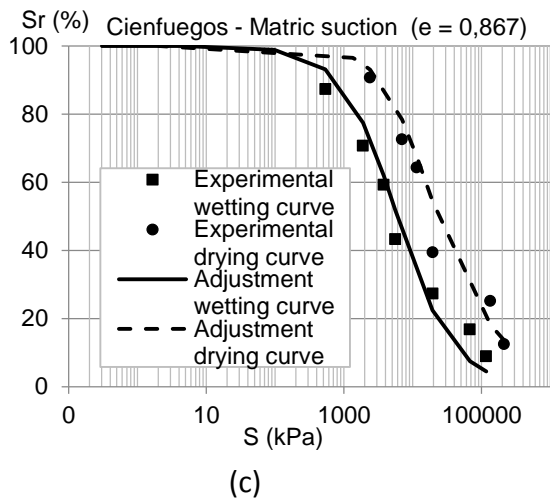
Comparison of the simulation of the characteristic curves for the models studied.

For adjustment of models with experimental values is assumed that the experimental value obtained is equal to the estimated value for the analyzed model, the pairs $\left(Y_i \,;\, \hat{Y}_1\right)$ must belong to the line y = x. Adjusting these pairs a simple linear regression model y = a + bx was used, where we should expect "a = 0" and "b = 1".

After calculating, the coefficients of the regression model are compared with the actual values of model. Besides the quality of fitness is assessed by the ANOVA model corresponding correlation coefficient, coefficient of determination (R^2), R^2 adjusted for degrees of freedom, the standard error of the estimate is obtained (error studentized) and mean absolute error. Table 5 appear parameters ANOVA model for each of the models and adjusting for each of the soils under study.

Table 5 Parameters of ANOVA Model for Mariel soils, Model of Van Genuchten. Source: own elaboration, 2013

Parameters	Model of Van Genuchten		Model of Fredlund		Model of Gallipoli	
	Curve in drying.	Curve in wetting.	Curve in drying.	Curve in wetting.	Curve in drying.	Curve in wetting.
correlation coefficient	0,9606	0,9710	0,9617	0,9599	0,9589	0,9723
R^2, coefficient of determination (%)	922,692	942,868	924,948	921,371	919,471	945,279
R^2 (adjusted for degrees of freedom) (%)	884,037	914,302	887,422	882,057	879,206	917,919
error studentized	0,0530	0,0423	0,0534	0,0641	0,0613	0,0379
mean absolute error	0,0309	0,0256	0,0312	0,0358	0,0356	0,0238
$\lvert a - \hat{a}\rvert = \lvert\hat{a}\rvert$	0,0188	0,0306	0,0110	0,0252	0,0346	0,0718
$\lvert b - \hat{b}\rvert = \lvert 1 - \hat{b}\rvert$	0,0758	0,1276	0,1003	0,1145	0,2157	0,1985

In the parameters $\lvert\hat{a}\rvert$ and $\lvert 1 - \hat{b}\rvert$, table 5, the absolute error in the estimated values of the coefficients of the regression model. This information is taken into account by the adjusted coefficient of determination, necessary to decide the quality of the model's fitness of the curve of best performance according to the experimental data obtained.

For soil Capdevila and Sagua la Grande the Gallipoli's model is the one that best fits to the experimental values; for the ground of Cienfuegos is the model of Van Genuchten and soil Mariel is model of Fredlund, on this soil as it is evidenced in table 5 was where the lowest values of R^2 (coefficient of determination) set the degree of freedom of the problem, this model was chosen because it is where the most stable values were obtained in the approximation For all of the cases were considered the absolute error in the estimation of the coefficients of the regression model and the coefficient of determination adjusted for degrees of freedom of the problem, to select the best fit model.

Following an assessment of the residue error curves for the models chosen in each soil it was found as result. The error residues curve for Mariel sample on the wetting branch in the Fredlund model best fitted for the points 1, 2 and for the drying branch points 1 and 2 are atypical residue values being greater than 2, they cannot be excluded from the experimental values to consider in the analysis because with only two values the sample is not representative, as the value of R2> 90% adjustment is considered acceptable.

For other studied soils analysis curves obtained residue retrieving the following ratings. In the curve of error residues for Capdevila in the branch of wetting the Gallipoli model fits better all of the values of atypical residue, however, in the branch on drying there is a value of atypical residue being greater than 2, it was excluded from the analysis of the experimental values.

For the soil of Sagua la Grande on the wetting branch the Gallipoli model was found of best fit, it was found a value of atypical residue being greater than 2, this point can be excluded and for drying branch is no atypical residue values; and sample from Cienfuegos on the wetting branch for Van Genuchten model was found to be the best fit for the branch and there was discovered a point on drying branch atypical residue values on each curve being greater than 2, they can be excluded from the experimental values to be considered in the analysis.

All the previous statement allow us to establish this procedure as adequate for de Cuban conditions, necessary to obtain the soil – water characteristic curve (Sr vs S), valid for the estimation of various soil parameters employed in the description of unsaturated soils.

Conclusions

The estimation of the soil-water characteristic curve has different interests and can be employed to obtain several parameters used in the description of unsaturated soils, where the knowledge of the soil-water characteristic curve is essential. Permeability, shear strength and volumetric behavior are soil properties that are affected by the variation of suction, can bring direct effects on the building project, obtaining the capacity and settlements produced in foundations built on these soils.

From the experimental results using the method of the filter paper has shown that by adjusting the methods of Van Genuchten, Fredlund and Gallipoli, a coefficient of determination R^2 > 95% is satisfactory in engineering; considering valid the employment of any of the three methods for adjusting the experimental curves obtained.

For the soil Capdevila and Sagua la Grande the model Gallipoli is the one that best fits the experimental values obtained in the laboratory, to the soil of Cienfuegos is the model of Van Genuchten and the soil Mariel is the model of Fredlund, in this case as it is evidenced in table 5, where resulted the lowest values of the coefficient of determination for the selected degree of freedom of the problem; it was chosen for each case the corresponding model according to the most stable values in the approximation.

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